



<b>Title:</b>	<b>The Phantastic Photon and LEDs (combined lab)</b>
<b>Original:</b>	8 July 2009
<b>Revision:</b>	27 June 2010
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<b>Appropriate Level:</b>	High School Physics
<b>Abstract:</b>	<p>This is actually two labs that work very well in combination. In the first, students investigate the relationships among color, frequency, energy, and wavelength of light using spectrosopes, colored LEDs, phosphorescent, and fluorescent materials. They observe characteristics, measure wavelengths, and observe and calculate photon energies.</p> <p>The second lab has students investigate the conversion of electrical energy into light and vice versa. It uses electrical measurements on super-bright LEDs to determine the energy of various frequencies (colors) of light. By graphically comparing the energy lost per electron with the frequency of the emitted light for several LED colors, students estimate Plank's constant, a fundamental value in quantum mechanics.</p>
<b>Time Required:</b>	Two 40 minute class periods
<b>NY Standards Met:</b>	<p>4.1a All energy transfers are governed by the law of conservation of energy.</p> <p>5.3c On the atomic level, energy is emitted or absorbed in discrete packets called photons.</p> <p>5.3d The energy of a photon is proportional to its frequency.</p>

**Xraise Outreach for CLASSE**

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**Special Notes:**

**The Phantastic Photon/LEDs** is a kit available from the CIPT Equipment Lending Library, [Xraise.classe.cornell.edu](http://Xraise.classe.cornell.edu). It is also available commercially from West Hill Biological Resources, Inc., [www.westhillbio.com](http://www.westhillbio.com).

Created by the CNS Institute for Physics Teachers via the Nanoscale Science and Engineering Initiative under NSF Award # EEC-0117770, 0646547 and the NYS Office of Science, Technology & Academic Research under NYSTAR Contract # C020071

## **Behavioral Objectives**

Upon completion of this lab, a student should be able to:

- define the terms: photon, discrete packet, excited state
- explain the basic concepts of the particle nature of light
- explain how energy, frequency, wavelength, and color of photons are related.
- describe the appearance of the spectra of LEDs
- solve basic problems using the formulas  $c = f \cdot \lambda$  and  $E = h \cdot f$
- compare and contrast phosphorescent and fluorescent materials, and explain the excitation of each by light photons.
- explain how distance from a light source influences illuminated of a surface and resulting excitation.
- describe the appearance of materials containing excited quantum dots, and state the size-color relationship of quantum dots.
- explain the reversible conversion of energy between electrical and light energy by LEDs
- describe the electrical energy lost and frequency of light emitted by different colored LEDs
- measure frequency and electrical energy data from LEDs, and analyze that data to estimate Planck's constant

## **Class Time Required:**

Three 40-minute periods.

## **Teacher Preparation Time:**

Prep time is 5-10 minutes. Set out materials needed.

## **Assumed Prior Knowledge of Students:**

- Relationship between frequency and wavelength ( $f = c/\lambda$ )
- Conservation of energy

## **Background Information for Teacher:**

Fluorescent and phosphorescent materials are all around us. They contain molecules that can absorb and release photons. When such a molecule absorbs a photon, gaining its energy, it enters an excited *molecular* state. At a later time, the molecule loses some of its energy by emitting a visible photon of lower energy and entering a lower molecular energy state. These emitted photons are what make glow-in-the-dark objects glow and fluorescent materials look very bright. The primary difference between fluorescent and phosphorescent materials is the mean time that a newly excited molecule spends in the excited state. Phosphorescent materials typically take much longer to emit photons from the excited state than fluorescent materials. Glow-in-the-dark (phosphorescent) objects continue to emit photons for minutes or even hours after exposure to

light while fluorescent molecules emit photons very quickly, within nanoseconds of absorbing energy from photons.

Phosphorescent molecules take a long time to return to the ground state because the excited electron has the 'wrong' spin. Recall that the Pauli Exclusion Principle forbids two electrons of the same spin to occupy the same state. If the excited electron has the same spin as the other electron in the lower energy level, it must flip its spin before it is allowed to occupy that energy level. Since interactions in which an electron flips its spin are very rare, the electron in the higher energy level has to wait a long time to lose its energy and give off a photon.

Why do fluorescent and phosphorescent materials emit photons of *lower* energy than what they absorb instead of emitting a photon at the same energy? The answer lies in the fact that some of the energy of the excited molecule is converted to a different (non-photon or lower energy photon) form. Some energy can go to a visible photon while another portion of the energy goes to a neighboring atom or molecule through atomic collisions. An excited molecule can also emit a low-energy infrared photon (not visible) and lose a small amount of energy, leaving it in an intermediate state which is at a lower energy than the original excitation caused.

### LEDs:

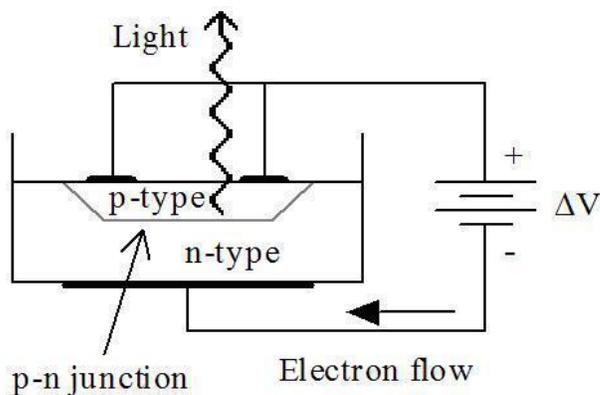


Figure 1. A light-emitting diode (LED).

Figure 1 shows the structure of a *light-emitting diode* (LED)—a device that emits light when electric current flows through it. Unlike a light bulb, an LED converts electrical energy into light energy at the interface between two different semiconductor materials called a *p-n junction*. The reason for this name is that one of the materials has a chemical composition that furnishes excess mobile electrons (the negative or *n-type* material), while the other has a different composition with a deficiency of mobile electrons (the positive or *p-type* material).

This lab explores the energy conversion:

**Electrical energy -> Photons (bundles of light energy)**

This lab is not a detailed investigation of the structure and mechanisms of p-n junctions; however the following simple physical model does help explain how the junction works. Some of the electrons at the junction diffuse across the interface from the n-type material to the p-type material. This leads to the situation, shown in Figure 2, where a slight excess of negatively charged ions (with bound electrons) appear on the p-side in the region close to the junction, and a slight excess of positively charged ions (missing electrons) appear on the n-side in the region close to the junction. The curve shows the potential energy of mobile electrons on both sides and in the junction region.

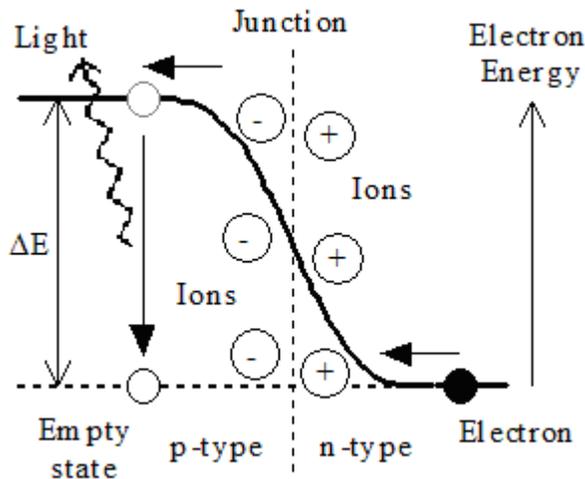


Figure 2. Operation of an LED

Mobile electrons on the p-side of the junction occupy energy levels that lie an amount  $\Delta E$  above empty energy states there. When a large enough external voltage difference  $\Delta V$  is applied to the junction with the proper polarity, mobile electrons from the n-side can be driven up the energy hill and across the interface to the p-side. When an electron finds an empty state there and falls into it, the electron loses energy  $\Delta E$ . This energy can then be carried off in the form of a *photon* of light, provided competing processes don't absorb the energy and that the p-type layer is thin enough for the light to escape. **The key idea here is that  $E_{\text{photon}} = |\Delta E_{\text{electron}}|$ .**

If the external voltage is applied to the LED with the opposite polarity, no electron current flows because electrons on the n-side are now driven away from the junction rather than across it. This one-way current flow leads to another important use of the p-n junction, the one-way *rectifying diode* often used to convert alternating current electricity to pulsating direct current electricity. The light-emitting diode or LED combines these two properties: Electric current can only flow across the junction in one direction, and when it does, the energy  $eV$  gained by an electron from the applied voltage  $V$  can be released as a photon of light. This simple model leads to the expectation that if we gradually increase the applied voltage  $V$ , light should appear when  $eV = E$ , and the frequency  $f$  of the light should follow the relationship  $E = hf$ , where  $h$  = Planck's constant.

One nice thing about LEDs is that the materials used to make the p-n junction determine the value of  $\Delta E$  and can be "tuned" to give different colors of emitted light. Put another way, the height of the energy hill ( $E$ ) depends on the materials used, and the color of emitted light depends on  $E$ . Red LEDs (and red lasers) are made from gallium arsenide (GaAs) with small concentrations of phosphorus impurity added. Blue LEDs are made from a different material.

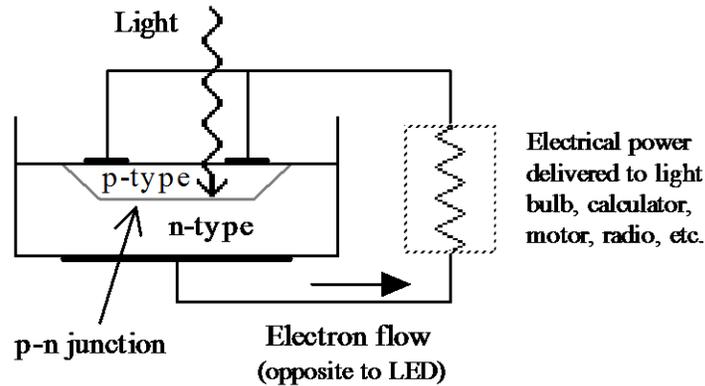


Figure 3. A solar cell.

The *solar cell* is a wonderfully variation on the LED. In a solar cell, light directed at the very thin top layer of a p-n junction penetrates to the junction interface. Here, if the photon energy ( $E_{\text{photon}} = hf$ ) is greater than or equal to the electron energy level difference ( $E$ ), then *bound* electrons on the p-side can absorb these photons and be freed. The freed electrons raise in energy by  $E_{\text{electron}} = E_{\text{photon}}$  to become *mobile* electrons which are then driven *down* the energy hill shown in Figure 2. This is opposite to the direction of electron flow that makes an LED light up. In other words, *a solar cell is an LED run backwards*. The electric current (called a "photocurrent") produced by a solar cell depends on the intensity of light illuminating it.

## Equipment



Item Number	Quantity	Item
1	1	Spectrometer
2	1	Phosphorescent tape
3	1	Quantum Dots
4	1	Power adapter (24V, 0.25A)
5	2	Alligator clip leads
6	1	LED board holder
7	1	Diffraction grating
8	1	LED board
9	1	Fluorescent paint card
Not pictured	2	Alligator clip leads
Not pictured	1	Red LED
Not pictures	1	Green LED
Not Pictures	1	Digital multimeter

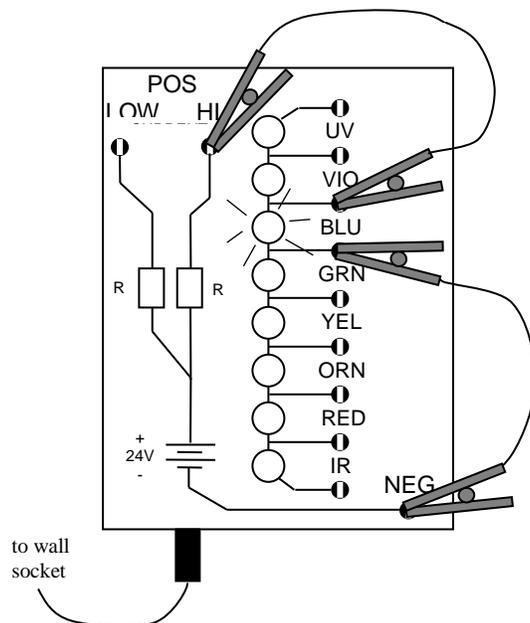
# THE PHANTASTIC PHOTON

## Introduction:

Why does ink from a fluorescent highlighter appear so bright? What makes glow-in-the-dark objects glow? How do optical brighteners in laundry detergent make your clothes look whiter? The answer to these and many other questions can be found in the photon theory of light, first proposed by Albert Einstein in 1905. By exposing fluorescent and glow-in-the-dark materials to different colors and intensities of light, you will see photons at work.

**Light Emitting Diodes (LEDs)** - You have the LED card shown below. There are eight LEDs of different colors, as labeled on the right side. The 'UV' (ultraviolet) and 'IR' (infrared) LEDs emit most of their light at wavelengths your eye cannot see. The other six LEDs have the different colors of the rainbow.

Your teacher will give you instructions on how to light the lights on the board. Refer to the illustration below as a reference.



- Experiment with lighting up other LEDs. Can you light more than one at a time?

**Disconnect a wire to turn off the LED card when you are not using it.**

**Warning!! Do not stare into the ultraviolet (UV) LED when it is lit. This can cause eye damage.**

### A. Glow-in-the-right color

A glow-in-the-dark object only glows after it is exposed to light. How does the color of the light affect a glow-in-the-dark object? You can test this with the LEDs and piece of glow-in-the-dark tape enclosed in black paper. Keep the tape covered when it is not in use.

- Hook up the blue LED using the “Hi” current setting.
- Shine the blue LED on the tape for a couple seconds.
- Move the LED away from the tape and see if the tape is glowing.
- Record your observation for the blue LED in the middle column of the table below (‘yes’ it makes the tape glow or ‘no’ it does not), and then test the other LED colors. If the tape glows, wait ~ 30 seconds before testing the next LED color to allow the tape to stop glowing.

LED color	Tape glows? (YES or NO)	LED wavelength (nm)
blue		
red		
green		
orange		
infrared		875 nm
violet		
yellow		
ultraviolet		395 nm

### B. Exploring the Wavelengths of Colors

- Disconnect the wires from the LED circuit board and slide it into the holder on the back of the spectrometer so that the LEDs face the slit.
- Reconnect the wires to light all the LEDs except the ultraviolet (U.V.) using the “HI” current positive terminal, as shown in the picture. The bottom wire connections are made through the holes in the bottom of the holder.
- Hold the spectroscope up and look through the diffraction grating (the slide mounted in the front). As you look towards the paper scale on either side of the LEDs, you should see the portion of the spectrum which is produced by each LED. If the bands of colors aren’t horizontal, rotate the diffraction grating slide  $\frac{1}{4}$  turn in the holder.

- Read the wavelengths of the six visible LEDs using the nanometer (nm) scales to the left and right of the LED strip. Since each LED produces a range of colors, use the center of the brightest color the LED produces. Record the wavelength you observe in the last column of the table in section A. (Hint: For some LEDs, the 'LOW' current setting will provide more accurate results.)

**Answer the following questions:**

1. What do you notice about the wavelengths of the LEDs that make the tape glow?
2. Light is a form of energy. Which wavelengths do you think contain the most energy? Explain.
3. Notice that the tape always glows the same color no matter what color activates it. Write the color that it glows \_\_\_\_\_.
4. Estimate the wavelength of light emitted by the tape \_\_\_\_\_.
5. How does the *wavelength* of the light emitted by the glowing tape compare with the *wavelength* of the LED light used to activate the tape?
6. Why do you think some of the colors of light did not activate the tape?

**C. This is intense!**

The light from an LED gets more intense as you move the LED closer. Is intensity an important factor in whether or not the glow-in-the-dark tape glows?

- With the lights dimmed, place a piece of paper so that it covers half of the glow-tape, blocking it from exposure to the LED light.
- Expose the tape for a few seconds to one combination of distance and LED color at a time, as outlined in the table below.
- After each trial, remove the paper and immediately examine the entire tape to see if the paper left a shadow. If the exposed part of the tape is glowing; write "Yes" in the space provided. If there is no difference, the exposed tape is not glowing; write "No."

<b>Distance</b>	<b>Blue LED</b>	<b>Yellow LED</b>	<b>Red LED</b>	<b>Ultraviolet LED</b>
<b>0.01 m</b>				
<b>0.10 m</b>				
<b>1.0 m</b>				

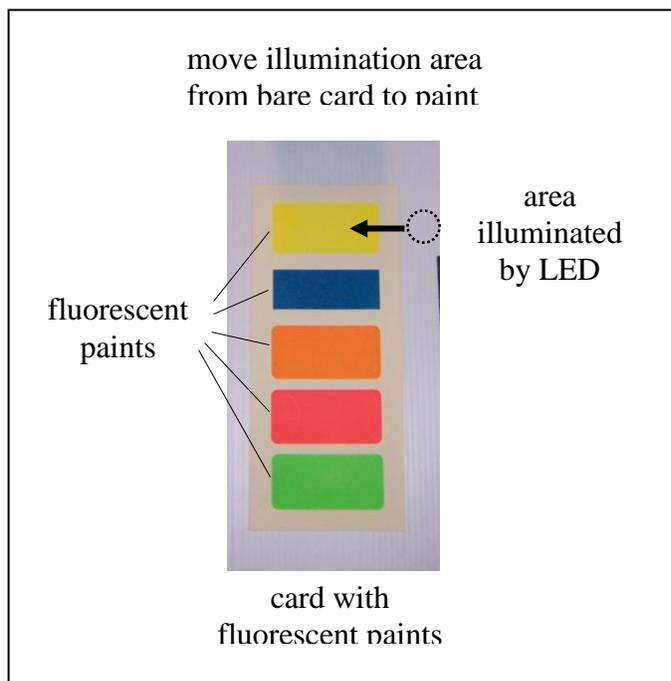
**Answer the following questions:**

7. Does the intensity of the LED light make a difference in *how brightly* the tape glows? Describe your results.
8. Does the intensity of the LED light make a difference in *whether* the tape glows or not? Describe your results.
9. Do you think the tape would glow if it received only a single particle of light from the ultraviolet LED? Explain.

**D. Look at What Popped Out!**

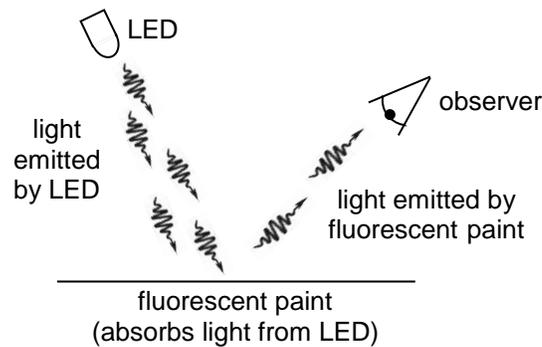
What makes fluorescent colors look so bright? Do fluorescent colors look bright in all kinds of light? You can test this with the LEDs and the manila card with different colors of fluorescent poster paint.

- Shine light from the UV LED on the fluorescent paints. Then shine light from the red LED on the fluorescent paints. Which light makes the paints "pop out" or look brighter?
- Let's take a closer look. Shine the light from the UV LED on the card where there are no paints. It should look violet. Now shine the light from the UV LED on the yellow fluorescent paint. The light should look yellow.
- Do the same thing with the red LED. Does the red light also change color on the yellow paint?
- Let's analyze this. Shine the light from the UV LED on the card where there are no paints. Take the spectrometer (without the LED holder) and aim it so that you can see a pool of violet light on the card when you look through the diffraction grating and the slit. You should see the diffracted light fall on the scale to either side of the slit.



- Now shine the light from the UV LED on the yellow fluorescent paint. Aim the spectrometer at the yellow paint where the LED illuminates it. Did the wavelength of the light change?
- Try the same thing with the red LED, analyzing the light from the card and from the paint with the spectrometer. Does the wavelength shift as it did for the UV LED?

The diagram below shows that the light from the LED gets absorbed by the fluorescent paint. Then the paint emits light, some of which reaches your eye.



**Answer the following questions:**

10. Predict the color of light that will be emitted from the yellow fluorescent paint for each LED. Write your predictions in the table below. Then test your predictions using the LEDs and the spectrometer to analyze the color from the paint. Record your results below.

LED color	Predicted color of light from yellow paint	Observed color of light from yellow paint
Red	--	red
Orange		
Yellow		
Green		
Blue		
Violet		
Ultraviolet	--	yellow

11. Why does the ultraviolet light get converted to yellow light by the yellow fluorescent paint, but the red light remains red?

12. White light is composed of all colors of light. Explain why white light makes yellow fluorescent paint look so intensely yellow.

13. Predict the color of light that will be emitted from the *different* fluorescent paints for a *green* LED. Write your predictions in the table below. Then test your predictions using the *green* LED and the spectrometer to analyze the color from the paint. Record your results below.

Paint color	Predicted color of light from paint	Observed color of light from paint
Red		
Orange		
Yellow		
Green		
Blue		

14. Explain your observations from the table above.

### E. Quantum Dots

Semiconductor nanocrystals or quantum dots are tiny nanometer-sized semiconductor particles (containing roughly 100 to 1000 atoms) that emit light when excited. Quantum dot diameters range from about 2 to 6 nm, which is about 40,000 times smaller than the diameter of human hair! The color of light emitted by these recently discovered structures depends on the diameter of the nanoparticles, with larger diameter particles emitting lower energy light. You have four different samples of CdSe quantum dots, each containing millions of nanoparticles of uniform size.

- Shine light from the UV LED on the four quantum dot samples. This will excite the quantum dots. What colors do the quantum dots emit?

- Now shine light from the RED LED on the quantum dot samples. Can you excite the quantum dots with red light?

**Answer the following questions:**

15. What colors of light did each of the four quantum dot samples emit?
16. Which quantum dots have the largest diameter? Which have the smallest? Why?
17. Predict which quantum dots will be excited by each LED color and write your predictions in the table below. Then test your predictions and record the results.

<b>LED color</b>	<b>PREDICTIONS: List the quantum dots you <u>expect</u> will be excited by each LED</b>	<b>OBSERVATIONS List the quantum dots that <u>are</u> actually excited by each LED</b>
<b>Red</b>		
<b>Orange</b>		
<b>Yellow</b>		
<b>Green</b>		
<b>Blue</b>		
<b>Violet</b>		
<b>Ultraviolet</b>		

18. Why can't the red LED excite the green quantum dots?
19. What determines whether a particular quantum dot will be excited by a particular LED?

**F. Post-lab Questions**

According to Einstein's theory, light is composed of tiny particles called "photons." A photon is the smallest possible amount of light. You can think of it as a really tiny

packet of energy. The energy of a single photon is proportional to the frequency of the light. If  $E$  is the energy of a single photon and  $f$  is its frequency, then

$$E = hf$$

where  $h$  is an extremely important physical constant called Planck's constant. Planck's constant is equal to  $6.6 \times 10^{-34} \text{ J}\cdot\text{s}$ .

20. Complete the chart below by calculating the energy of a single photon of light for each of the LEDs in your set. Remember that the frequency of light  $f$  is related to its wavelength  $\lambda$  through the formula  $f = c/\lambda$  where  $c$  is  $3.0 \times 10^8 \text{ m/s}$ .

LED color	Wavelength (nm)	Wavelength (m)	Frequency (Hz)	Photon energy (J)
infrared				
red				
orange				
yellow				
green				
blue				
violet				
ultraviolet				

21. As wavelength increases, what happens to the energy of a photon?

22. As the number of photons increases, what happens to the total energy of the light?

23. The red LEDs used here convert most of their  $\sim 0.03\text{W}$  of power into light. ( $1 \text{ W} = 1 \text{ J/s}$ ) Estimate the number of photons per second produced by the red LED.

24. Use the concept of photons to explain why red light, even if it is intense, cannot make the glow-in-the-dark tape glow (emit light).

25. When the glow-in-the-dark tape absorbs blue photons, it emits lower energy yellow-green photons. If energy is always conserved, explain how the energy emitted can be less than the energy absorbed.

26. Use the concept of photons to explain why a yellow fluorescent highlighter appears much brighter than a regular yellow marker in normal lighting conditions.

27. Some clubs use black lights (ultraviolet lights) for a special effect to make white clothing glow. Explain how this works.

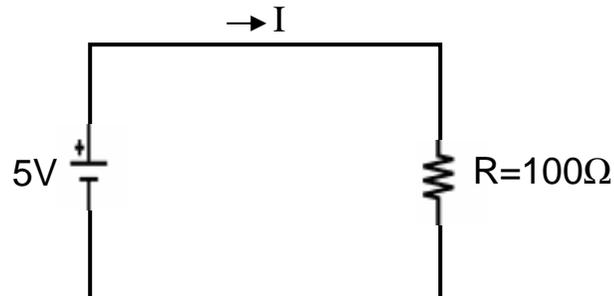
28. Photoresist, a chemical used in making computer chips, changes its solubility when exposed to ultraviolet light. Why are cleanrooms where photoresist is used illuminated with yellow light?

29. A silicon photodiode used as a light detector can only absorb photons of energy greater than 1.1 eV. Will it absorb photons from the infrared LED?  
(Hint:  $1.0 \text{ J} = 1.6 \times 10^{-19} \text{ eV}$ )

## LIGHT EMITTING DIODES

### A. Prelab

Consider the following circuit with a 5 Volt battery and 100  $\Omega$  resistor:



1. What is the relationship between electric potential and electric potential energy?
2. How much electric potential energy does each electron lose when passing through the resistor?

Now consider light of frequency ( $f$ ), wavelength ( $\lambda$ ) and velocity of light ( $c$ ).

3. What is the relationship between  $f$  and  $\lambda$ ?
4. What is the relationship between the energy of a photon ( $E$ ) and frequency of its light ( $f$ )?
5. Calculate the energy of a photon of wavelength 500 nm. ( $1\text{nm} = 10^{-9}\text{m}$ )
6. If electrons flowing in a circuit were to lose an amount of energy equal to the photon energy found in the last question, what would be their change in voltage?

**Introduction:**

Light Emitting Diodes (LEDs) are in use everywhere you look! They are found in digital clocks, TVs, VCRs, coffee makers, cell phones, computers, digital books, etc., etc. LEDs emit light by converting electrical potential energy into light energy, a property which makes LEDs useful as indicators. The color of light that an LED emits depends on its material composition. By lighting up LEDs of different colors and measuring some of their electrical and optical properties, you will estimate the value of Planck's constant.

**B. Making LEDs Light Up**

7. Try hooking up the positive and negative leads both ways to the LED. Does it matter which way you hook them up?
8. What is the voltage across the red LED when it is lit?
9. What is the voltage across the blue LED when it is lit?
10. Based on your voltage measurements, do electrons in the red LED or the blue LED lose the most energy?
11. Which color of light contains photons of greater energy, red or blue?
12. What is the qualitative relationship between voltage drop across an LED and the color of light it emits?

**C. LEDs in Reverse**

You will need one red LED, one green LED, and one LED card for this activity.

- Light the green LED on the LED card using "HI CURRENT" terminal.
- Place a voltmeter across the terminals of the individual red LED.
- Orient the two LEDs so that light from the green LED can shine directly into the red LED. Make sure the axes of the two LEDs are aligned and parallel.
- Adjust the two LEDs until you find the maximum voltage across the red LED with green light shining into it.

13. What is the maximum voltage?
14. What is the voltage across the red LED when the green LED is turned off?
15. Explain the last two voltage measurements.

Now do the reverse experiment.

- Light the red LED on the LED card using "HI CURRENT" terminal.
- Place a voltmeter across the terminals of the individual green LED.
- Orient the two LEDs so that light from the red LED can shine directly into the green LED. Make sure the axes of the two LEDs are aligned and parallel.
- Adjust the two LEDs until you find the maximum voltage across the green LED with red light shining into it.

16. What is the voltage across the green LED with the red light shining into it?

17. Explain your observations from the previous question.

18. Predict what will happen if you shine ultraviolet light into a green LED.

19. Try the experiment from the last question and describe your results.

#### D. Measuring Planck's Constant

- Copy the frequencies you calculated in question #20 of the first part of this lab (The Phantastic Photon) from that table into the table below. Do **not** copy the energy ... it will be determined experimentally in a moment.
- Light all eight LEDs using the "LO CURRENT" terminal. You should see most of the LEDs emit barely visible light.
- Measure the voltage across each LED and write it in the table below.
- Use the voltage to calculate the energy lost by each electron when it passes through the LED and enter it in the "Energy lost" column.
- Convert the energy values in Joules to units of eV ( $1\text{eV} = 1.6 \times 10^{-19}\text{J}$ ).

LED	Frequency (Hz)	LED voltage (V)	Energy lost (J)	Energy lost (eV)
infrared				
red				
orange				
yellow				

green				
blue				
violet				
ultraviolet				

20. Compare the voltages of the different LEDs with the frequency of the light emitted by the LEDs. What trend do you observe? Explain.

Plot the **energy lost by an electron versus the frequency of the light emitted** for all the LEDs on a separate piece of graph paper or on a computer.

21. What does the slope of this graph represent?

22. Find the slope of the graph and record it here with units included.

23. Why does this experimental value differ from the established value of  $6.626 \times 10^{-34}$  J·s?

**Max Planck** deduced that the relationship between the energy and the frequency of electromagnetic radiation is  $E = hf$ . He announced his derivation of the relationship in a paper published in 1900, and received the Nobel Prize for this work in 1918.

**Albert Einstein** commented on Max Planck's discovery in "On Quantum Physics" in 1954 as follows:

"In the year nineteen hundred, in the course of purely theoretical (mathematical) investigation, **Max Planck** made a very remarkable discovery: the law of radiation of bodies as a function of temperature could not be derived solely from the Laws of Maxwellian electrodynamics. To arrive at results consistent with the relevant experiments, radiation of a given frequency  $f$  had to be treated as though it consisted of energy atoms (photons) of the individual energy  $hf$ , where  $h$  is **Planck's** universal constant. This discovery became the basis of all twentieth-century research in physics and has almost entirely conditioned its development ever since. Without this discovery

it would not have been possible to establish a workable theory of molecules and atoms and the energy processes that govern their transformations. Moreover, it has shattered the whole framework of classical mechanics and electrodynamics and set science a fresh task: that of finding a new conceptual basis for all physics. Despite remarkable partial gains, the problem is still far from a satisfactory solution.”

Expressed in the SI units of joule seconds (J•s), **Planck’s constant,  $6.626 \times 10^{-34}$  J•s, is one of the smallest constants used in physics.** This is significant because it reflects the extremely small scale at which quantum mechanical effects are observed, and the reason we don’t notice quantum physics in our everyday lives the way we notice classical physics all around us. Indeed, **classical physics can essentially be defined as the limit of quantum mechanics as the Planck constant tends to zero.**



Max Planck (1858-1947)